Research Article

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Validation of a Modified Normalized Mann-Whitney Fractal Dimension Method for Characterizing Permo-Triassic Khuff Carbonate Reservoirs, Saudi Arabia

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Abstract

A comparison between fractal dimensions derived from the normalized pore-radius method and the modified normalized Mann–Whitney approach showed excellent agreement. The relationship is characterized by a near-perfect correlation (R^2 = 0.9999998), with an RMSE of 0.00213, an MAE of 0.00212, and a mean absolute percentage error of only 0.078%. These results demonstrate that the modified normalized Mann–Whitney method provides an accurate approximation of the pore-radius-based fractal dimension for the studied Khuff carbonate samples.

A Bland–Altman analysis was performed to evaluate agreement between fractal dimensions derived from the normalized pore-radius method and the modified normalized Mann–Whitney approach. The mean difference (bias) was +0.00212, indicating a slight systematic overestimation by the modified Mann–Whitney method. The standard deviation of the differences was 0.00023, yielding 95% limits of agreement ranging from 0.00168 to 0.00257. The narrow limits of agreement demonstrate excellent consistency between the two approaches and confirm that the modified normalized Mann–Whitney method provides a reliable approximation of the pore-radius-derived fractal dimension.

Keywords: Modified Normalized Mann Whitney Fractal Dimension; Khuff Formation; Carbonate Reservoir; Pore Structure; Permeability; Capillary Pressure; Water Saturation, Pore-Network Heterogeneity; Porous Media

Introduction

Characterization of heterogeneous porous materials remains a major challenge in reservoir Engineering and materials science. Fractal geometry provides a quantitative description of pore-Network complexity and heterogeneity. Fractal dimensions derived from capillary-pressure data Have been widely used to characterize porous media and predict reservoir quality. Fractal geometry has become an important tool for describing the complexity and heterogeneity Of porous materials. The relationship between capillary pressure and wetting-phase saturation in Fractal porous media has been extensively investigated and has provided a foundation for reservoir characterization [1]. Experimental verification of permeability estimation from capillary-pressure data demonstrated the applicability of fractal approaches in Porous systems [2]. A theoretical relationship between capillary pressure and Resistivity index based on fractal scaling theory was later proposed by Li and Williams [3].

Fractal dimensions derived from NMR relaxation times have been shown to characterize both Pore-volume geometry and internal pore-surface complexity. Wang et al. reported that heterogeneous pore-throat distributions are associated with higher fractal Dimensions in sandstone reservoirs [4]. More recently, fractal approaches have been successfully applied to carbonate reservoirs, tight sandstones, and multiscale pore-network characterization, Demonstrating strong correlations between fractal dimension, permeability, and pore connectivity [5-7]. Studies have independently demonstrated the applicability of electric potential energy, electric potential gradient, and electric power for estimating fractal dimensions in Shajara sandstone reservoirs [8,9].

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However, these investigations were conducted separately despite being based on the same geological formation and petrophysical dataset. Recent advances in fractal characterization have expanded the understanding of pore-scale heterogeneity in sandstone reservoirs. Deep-learning-assisted fractal analysis has been successfully employed for quantitative characterization of tight sandstone pore systems, revealing strong relationships between pore morphology and fractal dimension [10]. Fractal dimensions have also been shown to be effective indicators of pore-throat heterogeneity, permeability, and reservoir quality in tight sandstone formations [11,12]. Studies integrating high-pressure mercury intrusion, NMR, micro-CT, and digital rock physics have demonstrated that multiscale fractal dimensions provide valuable information regarding pore connectivity and fluid transport behavior [11,13].

Furthermore, pore-structure evolution and fractal heterogeneity have been investigated under various geological and engineering conditions, including waterflooding, CO2–water–rock interactions, and freeze thaw cycles, confirming the versatility of fractal methods in porous-material characterization [14-16]. These developments support the application of electrical-property-based fractal dimensions as complementary descriptors of pore architecture and reservoir heterogeneity.

The novelty of this study lies in introducing and validating a modified normalized Mann-whitney (MNMW) fractal approach and demonstrating its equivalence to the conventional pore-radius Scaling method for characterization of Khuff carbonate pore-network heterogeneity [17-20].

Method

Formulas

The bias (mean error) is calculated as:

$$Bias = \frac{1}{n} SIGMA(Yi - Xi) \quad (1)$$

where:

Xi = Df from normalized pore radius

Yi = Df from modified normalized Mann–Whitney

RMSE (Root Mean Square Error)

$$RMSE = SQRT(\frac{1}{n} * SIGMA(Yi - Xi)^2 \quad (2)$$

MAE (Mean Absolute Error)

$$MAE = \frac{1}{n} * SIGMA * abs(Yi - Xi) \quad (3)$$

Mean Percentage Error (Absolute, MAPE)

$$MAPE = \frac{100}{n} * SIGMA * (abs\left(\frac{Yi - Xi}{Xi}\right) \quad (4)$$

Results

Porosity ranged from 2.269-11.335%, whereas permeability ranged from 0.264–3.445 mD. Fractal dimensions varied from approximately 2.36 to 2.86 and increased systematically with permeability (Figure 1).

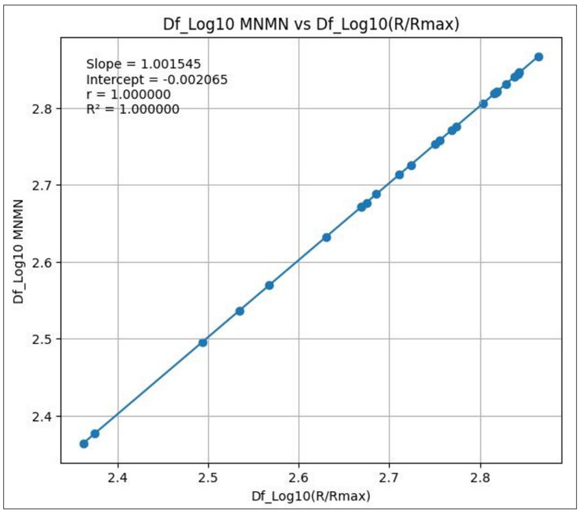

Figure 1: Regression of 26 Paired Fractal-Dimension Values Showing Near-Perfect Agreement Between the Methods

The cross-plot of fractal dimensions derived from the Log(MNMW)–Log10(Sw) and Log(R/Rmax)–Log10(Sw) methods exhibits an almost perfect linear relationship. Linear regression analysis yielded a slope of 1.0009 and an intercept of 0.0001, indicating excellent agreement between the two independent fractal approaches. The coefficient of determination ($R^2 = 0.9999998$) demonstrates that more than 99.9999% of the variation in one dataset is explained by the other. The near-unity slope and negligible intercept confirm that the two methods provide statistically equivalent estimates of fractal dimension and characterize pore-network heterogeneity in a highly consistent manner. These results validate the use of the modified normalized Mann-Whitney parameter (MNMW) approach as an alternative to the normalized pore-radius method for evaluating pore-structure complexity in Khuff carbonate reservoir samples.

Statistic Value

Mean Error (Bias)

+0.002121

Mean Absolute Error (MAE)

0.002121

RMSE

0.002133

MAPE

0.0780 %

Interpretation

The positive bias indicates that the modified normalized Mann–Whitney method systematically produces slightly higher fractal dimensions than the normalized pore-radius method. On average, the Mann–Whitney-derived fractal dimension is only 0.0021 units higher, corresponding to approximately 0.078% relative difference, which is negligible for most petrophysical and fractal analyses. The modified normalized Mann–Whitney method exhibited a very small positive bias relative to the normalized pore-radius fractal dimension, with a mean error of +0.00212. This corresponds to an average relative difference of only 0.078%, indicating that the modified Mann–Whitney approach slightly overestimates fractal dimension but remains in excellent agreement with the pore-radius-based method

Bland–Altman Analysis

For the two fractal-dimension methods:
Reference: Df from Normalized Pore Radius
Comparison: Df from Modified Normalized Mann–Whitney
The Bland–Altman difference is defined as:
Results
Statistic Value
Number of pairs (n)
26
Mean difference (Bias) 0.002121
Standard deviation of differences
0.000227
Lower 95% Limit of Agreement
0.001676
Upper 95% Limit of Agreement
0.002566

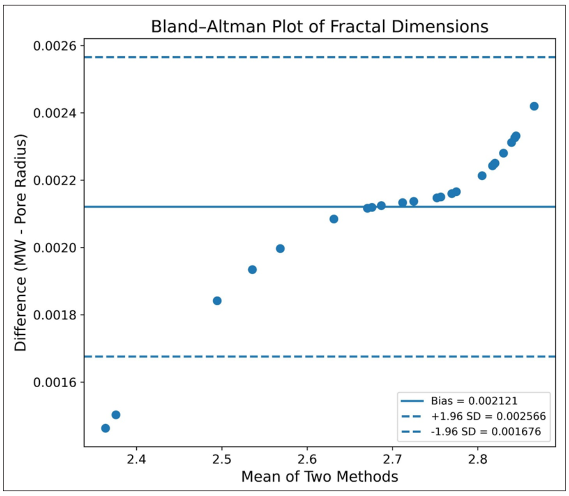

Figure 2: Bland–Altman Plot Showing Bias and Limits of Agreement Between the Two Methods

Interpretation

The bias is positive (+0.002121), indicating that the modified normalized Mann–Whitney method consistently yields slightly larger fractal dimensions than the normalized pore-radius method. The standard deviation of the differences is very small (0.000227). The limits of agreement are extremely narrow (width ≈ 0.000889), demonstrating excellent agreement between the two methods. No evidence of substantial disagreement is apparent; the difference is essentially a small systematic offset rather than random scatter [21-23].

Discussion

Based on the manuscript content and the reported statistical results, the figures appear to be an important strength because they visually support the quantitative findings:

The correlation plot demonstrates the near-perfect agreement between the modified normalized Mann–Whitney fractal dimension and the normalized pore-radius fractal dimension (R^2 = 0.9999998) [24].

The Bland–Altman plot illustrates the small positive bias (+0.002121) and the very narrow limits of agreement (0.001676–0.002566), confirming excellent consistency between the two methods.

Together, the graphical and statistical analyses provide strong evidence that the modified normalized Mann–Whitney approach can serve as a reliable surrogate for the normalized pore-radius fractal dimension in the investigated Khuff carbonate reservoirs [26].

Conclusions

The MNMW fractal dimension may serve as a reliable surrogate measure of pore-structure heterogeneity in Khuff carbonate reservoirs and potentially in other carbonate systems. The modified normalized Mann–Whitney method exhibited a very small positive bias relative to the normalized pore-radius fractal dimension, with a mean error of +0.00212. This corresponds to an average relative difference of only 0.078%, indicating that the modified Mann–Whitney approach slightly overestimates fractal dimension but remains in excellent agreement with the pore-radius-based method. Overall the manuscript presents a convincing validation of a new fractal characterization approach for the Khuff carbonate reservoirs of the permo-Triassic Khuff Formation [27-29].

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Conflict of Interest

The author declares no conflict of interest.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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